AIAA JOURNAL
Vol. 36, No. 8, August 1998

Practical Procedures for Reliability Estimation of Spacecraft Structures and Their Components

Gerhard S. Székely*
Leopold-Franzens University, A-6020 Innsbruck, Austria
Wolfgang H. Teichert†
Atos Netherlands BV, Leiden NL-2332 KG, The Netherlands
Christoph E. Brenner‡ and Helmut J. Pradlwarter§
Leopold-Franzens University, A-6020 Innsbruck, Austria
Michel Klein¶
European Space Agency, Noordwijk NL 2200 AG, The Netherlands
and
Gerhart I. Schuëller**
Leopold-Franzens University, A-6020 Innsbruck, Austria

The analysis of structural systems is mostly based on the assumption that the parameters are fixed, i.e., deterministic values. This contradicts the experience that the structural parameters often may vary (randomly) considerably. A way to model these uncertainties is to apply probabilistic methods. Two concepts are presented: The first is an engineering tool, which basically performs (weight-controlled) Monte Carlo simulation strategies using the capabilities of highly developed finite element programs, e.g., ASKA. A second concept is the stochastic finite element method using random fields that, combined with the response surface method, is a versatile tool to calculate the probabilities of failure. This approach is coded within the software package COSSAN, a tool for computational stochastic structural analysis. The feasibility of the concepts is presented using examples from spacecraft design, e.g., the influence of randomly distributed parameters on serviceability of devices mounted on satellites or the influence of randomly varying thickness on the reliability of an axially mounted fuel tank. Both concepts are shown to be practicable, especially for the design engineer.

Nomenclature

a, b, c	= parameters of the response surface for the limit state function
$\boldsymbol{C}_{ZZ}$	= correlation matrix of Z
D_f	= failure domain
$\boldsymbol{F}$	= external force vector
$F_X(x)$	= cumulative distribution function
$f_X(x)$	= probability density function
$g(\boldsymbol{X})$	= limit state function
$H, \boldsymbol{H}$	= random field, corresponding random variables from discretization
I	= indicator function
$\boldsymbol{K}$	= stiffness matrix
$\boldsymbol{M}$	= mass matrix
n_{int}	= number of integration points
n_{modes}	= number of retained modes
n_p	= number of points to set up the response surface
n_{rv}	= number of random variables
n_{sim}	= number of simulations within a Monte Carlo simulation
p_f	= probability of failure
$\boldsymbol{R}$	= linear, nonlinear restoring forces
R_{HH}, l_H	= autocorrelation function of random field and correlation length
S, s, X, x, Y, y, Z, z	= random variables and their realizations
t	= thickness
$\boldsymbol{u}$	= displacement vector
α	= standardized random variable
Γ, γ_i	= eigenvectors of the reference system
Θ, θ_i	= eigenvectors in the reduced space
$\lambda, \phi, (\Phi)$	= eigenvalues, eigenvectors (modes) of the physical eigenvalue problem
ξ, η, ζ	= spatial coordinates
Φ_G	= standard normal distribution
Ψ	= eigenvectors of the correlation matrix

Received Sept. 24, 1997; revision received April 10, 1998; accepted for publication May 5, 1998.

*Research Associate, Institute of Engineering Mechanics, Technikerstr. 13. Affiliate Member AIAA.
†Consultant, Haagse Schouwweg 8g.
‡Research Associate, Institute of Engineering Mechanics, Technikerstr. 13; currently Staff Engineer, Technologie Zentrum Steyr, Steyr-Daimler-Puch AG, Schönauerstr. 5, A-4400 Steyr, Austria.
§Associate Professor, Institute of Engineering Mechanics, Technikerstr. 13.
¶Head of the Structures Section, Mechanical Engineering Department, European Space Research and Technology Centre, Postbus 299.
**Chair of Engineering Mechanics, Institute of Engineering Mechanics, Technikerstr. 13.

I. Introduction

THE analysis of structures becomes increasingly complex. In early times, structures were reduced to simple mechanical models, e.g., beam models for bridges. The demand for more accuracy and the development of computers gave way to the possibility for the analysis of more complex structures, e.g., aerospace structures, tall buildings, etc. However, all of these calculations are based on the assumption that the loading as well as the structural properties can be considered as deterministically fixed values.

When approaching the limit states of a structure, it was recognized that these assumptions may no longer be maintained. Consequently, considerable efforts have been made to allow for random properties of loads, material, and geometry. These random properties can be described either in the form of single random variables or as random fields. The assumption that loading and structural parameters can be described statistically leads to results that are also randomly distributed. Often efforts are directed toward the computation of the statistics of the system response in terms of statistical moments or

sometimes distribution functions. Because of the complexity of this field, analytical studies are generally restricted to small systems. For systems with more than a few degrees of freedom, numerical analyses are necessary. However, these studies not only should allow for the estimation of the statistical properties of the structural response, e.g., displacements, stresses, etc., but also may be used to evaluate events with low probability, i.e., the probability of failure as an estimate for the reliability of a structure.

A most practical implementation[1] utilizes commercial finite element (FE) software, e.g., ASKA.[2,3] Here the random properties are given in terms of user-defined random variables that describe important parameters of the FE model. Each random variable describes a particular component of the model, e.g., the thickness or Young's modulus of elements, etc. For a direct Monte Carlo simulation (MCS), a preprocessing step provides random sets that are used in the FE analysis part to compute the structural response. The results are then visualized by a postprocessor. A special method is developed to reduce the system size in such a way that the MCS procedure is particularly suitable for dynamic analysis of large FE structures, such as spacecraft structures, etc.

Another most useful approach to determine the reliability of engineering structures with possible dynamic and nonlinear behavior uses the so-called stochastic finite element method (SFEM), which is based on the random field concept. For an overview, see Refs. 4–7. The procedure as suggested here employs the response surface method (RSM) (see Refs. 8 and 9) as a tool to determine the limit state function and consequently the probability of failure. In this context note that the effort to compute points on the limit state, which are subsequently used by the RSM, increases quadratically with the number of random variables. Therefore, transformations to the so-called uncorrelated normal space and, where necessary, sensitivity analyses, are proposed to identify those random variables that are expected to have a significant influence on the structural response. This approach is realized within COSSAN, a multipurpose and user-friendly code for computational stochastic structural analysis.[8]

For the analysis of extreme events and/or in case of the usage of materials and components with (highly) nonlinear restoring-force characteristics, it is important to use suitable analysis models. For example, ultimate failure might occur only after plastic deformation, or special damping devices might react only in extreme cases. In those cases nonlinear models must be considered in the analysis, and hence the numerical computation of results is highly time consuming. On the other hand, for structures modeled by finite elements, in many cases linear analysis procedures already yield sufficient insight to the problem. Therefore, in this paper only examples of linear systems are investigated, although both approaches allow for nonlinear analyses as well. These examples show the applicability and respective advantages of the approaches as presented.

II. Methods of Analysis

A. Direct Monte Carlo Simulation and Importance Sampling

A possibility to take into account uncertainties in a system is to model important parameters in terms of random variables X_i. They are statistically defined by their probability density function (PDF) $f_{Xi}(x_i)$ or the cumulative distribution function (CDF),

$$F_{Xi}(x_i) = \int_{-\infty}^{x_i} f_{Xi}(\chi)\,\mathrm{d}\chi \tag{1}$$

respectively.

The structural analysis can be regarded as a function $r(X_1, \ldots, X_{n_{\mathrm{rv}}})$, which maps the random variables X_i to the result $Y = r(X_1, \ldots, X_{n_{\mathrm{rv}}})$. Thus, it can be seen clearly that every result Y yielded by the analysis is also of a statistical nature and can be described by its PDF or CDF, respectively. Because of the complexity of the function r in the case of reasonably complex problems, it is generally impossible to find an analytical solution for the PDF or CDF of the results Y. Therefore, a numerical evaluation is needed.

The most direct way to perform this task is to apply MCS. For n_{sim} simulations, realizations x_{ij} with $i = 1, \ldots, n_{\mathrm{rv}}$; $j = 1, \ldots, n_{\mathrm{sim}}$ of the random variables X_i are generated, taking into account their distribution and correlation. To each realization $\boldsymbol{x}_j = (x_{1j}, \ldots, x_{n_{\mathrm{rv}}j})$, a corresponding result $y_j = r(\boldsymbol{x}_j)$ is calculated.

The objective of a statistical analysis is to obtain the distribution CDF $F_Y(y)$ of the result Y. Numerically this can be accomplished with

$$F_Y(y) = \frac{1}{n_{\mathrm{sim}}} \sum_{j=1}^{n_{\mathrm{sim}}} I(y - y_j) \tag{2}$$

where I is an indicator function, such that $I = 1$, if its argument is greater than zero, and $I = 0$ elsewhere.

An inherent drawback of Monte Carlo procedures is the associated statistical uncertainty of the estimates due to the finite sample size. One obvious way of reducing statistical uncertainty is to increase the sample size. However, in many cases this may not be very economical because there is a square root relationship (with respect to the variance) between the estimated error and the sample size. A way to circumvent an impractical or infeasible sample size is to change or distort the original problem in such a manner that the uncertainty in the estimate of the result is reduced in the area of major interest. Such procedures are called variance reduction techniques.[10,11] For example, the widely applied so-called importance sampling technique uses a sampling density function $f_h(x)$ that might differ considerably from the original function $f_X(x)$. The objective is to concentrate the distribution of the sample points in the domain of major importance. This can be done without distorting the original estimate by modifying the weight of the realization by $f_X(x)/f_h(x)$. For example, low probability quantities such as failure probabilities usually are estimated with the following unbiased estimator:

$$p_f = \frac{1}{n_{\mathrm{sim}}} \sum_{j=1}^{n_{\mathrm{sim}}} I(y_j, D_f) \tag{3}$$

$$I(y_j, D_f) = \begin{cases} 1 & \text{for} \quad y_j \in D_f \\ 0 & \text{else} \end{cases} \tag{4}$$

More advantageously they are determined by importance sampling:

$$p_f = \frac{1}{n_{\mathrm{sim}}} \sum_{j=1}^{n_{\mathrm{sim}}} \frac{f_X(x_j)}{f_h(x_j)} \cdot I(y_j, D_f) \tag{5}$$

where the sampling density $f_h(x)$ is selected such that a considerable part of the generated sample falls within the failure domain D_f. The joint probability density functions can hereby be determined using the Nataf model.[12,13] However, it is not a straightforward task in all cases to determine a suitable, i.e., optimal, sampling density function.[14] Especially in cases where the relation between the input vector $\boldsymbol{X}_j$ and the response vector $\boldsymbol{Y}_j$ is not clear cut, it may be rather difficult to establish a suitable sampling density. This difficulty can be avoided, at least partially, by utilizing an adaptive importance sampling procedure.[15] The efficiency of the adaptive sampling procedure depends very much on the number of random variables involved; in other words on the dimension of the vector $\boldsymbol{x}$. In realistic problems, considerable effort is required to keep the number of random variables as small as possible. Sensitivity analyses and parameter studies may be used to achieve this goal.

B. Uncertainty Modeling Utilizing Commercial Finite Element Packages

To utilize commercial FE packages in an MCS without significant changes in the internal structure of the FE analysis part, three steps have to be performed: 1) generation of samples (preprocessing), 2) loop of n_{sim} distinct FE analysis runs, and 3) evaluation of results (postprocessing).

These steps are implemented in almost independent programs and organized by a surrounding shell. Note that the FE data are simply modified for the MCS. This has the advantage that they do not have to be exported through an interface, which is often difficult, especially for larger structures.

It is clear that the aforementioned way of direct MCS is a very time-consuming procedure. As the statistical error of the simulation decreases only with an increasing number of simulations, for a high accuracy of the results, many simulations, i.e., FE computations, are necessary. To reduce the extremely time-consuming computational effort, either the simulation part has to be reformulated such that

only the region of interest is covered, or the computation of the finite element part has to be accelerated. The first idea leads to advanced MCS, as described earlier.

A concept for the reduction of computer time within the FE part in the case of linear dynamic analysis is indicated in Ref. 16. The problem of interest is how to reduce the computational time to solve repeatedly the eigenvalue problem of linear dynamics,

$$\boldsymbol{K}_j \phi_{kj} = \lambda_{kj} \boldsymbol{M}_j \phi_{kj} \tag{6}$$

where ϕ_{kj} and λ_{kj} are the kth eigenvector and eigenvalue of the jth simulation realization, respectively.

To avoid the repeated setup of the matrices, one may assume that the influence of the random variables on the mass $\boldsymbol{M}$ and stiffness $\boldsymbol{K}$ matrices can be approximated by a first-order Taylor series, i.e.,

$$\boldsymbol{M}_j = \boldsymbol{M}_0 + \sum_{i=1}^{n_{\text{rv}}} \alpha_{ij} \, \Delta \boldsymbol{M}_i \tag{7}$$

with

$$\boldsymbol{M}_0 = \boldsymbol{M}(\mu_{Xi}), \qquad \Delta \boldsymbol{M}_i = \sigma_{Xi} \cdot \left. \frac{\partial \boldsymbol{M}}{\partial X_i} \right|_{\mu_{Xi}}, \qquad \alpha_{ij} = \frac{x_{ij} - \mu_{Xi}}{\sigma_{Xi}} \tag{8}$$

where $\boldsymbol{M}_0$ is the reference mass matrix, obtained using the mean values μ_{Xi} of the considered random variables X_i, and $\Delta \boldsymbol{M}_i$ is the differential matrices. The relations for the stiffness matrix are analogous.

A major reduction in computational time can be achieved if these truncated Taylor series are transformed to a reduced, generalized space. This is possible if we assume that only the first few modes are of interest. For this purpose, the eigenvalue problem of the reference system is solved for the first n_{modes} modes, leading to a truncated eigenvector matrix $\Gamma = [\gamma_1, \ldots, \gamma_{n_{\text{modes}}}]$. By pre- and postmultiplication of the series with this truncated matrix, i.e., $\hat{\boldsymbol{M}} = \Gamma^T \boldsymbol{M} \Gamma$, one obtains the reduced matrix

$$\hat{\boldsymbol{M}}_j = \hat{\boldsymbol{M}}_0 + \sum_{i=1}^{n_{\text{rv}}} \alpha_{ij} \, \Delta \hat{\boldsymbol{M}}_i \tag{9}$$

Because Γ is the mode matrix of the reference system ($\boldsymbol{K}_0$ and $\boldsymbol{M}_0$), the preceding multiplication results in diagonal matrices $\hat{\boldsymbol{K}}_0$ and $\hat{\boldsymbol{M}}_0$. However, the differential matrices $\Delta \hat{\boldsymbol{M}}_i$ and $\Delta \hat{\boldsymbol{K}}_i$ are not diagonal. Thus in each simulation realization the reduced eigenvalue problem

$$\hat{\boldsymbol{K}}_j \theta_{kj} = \lambda_{kj} \hat{\boldsymbol{M}}_j \theta_{kj}, \qquad k \in [1, \ldots, n_{\text{modes}}] \tag{10}$$

has to be solved, resulting directly in the eigenvalues λ_{kj} of the system. The eigenvectors θ_{kj} are assembled in the matrix Θ_j. The modes Φ_j of the jth simulation are simply obtainable by the back transformation

$$\Phi_j = \Gamma \Theta_j \tag{11}$$

Note that for large systems the solution of the eigenvalue problem is computationally the most crucial part of the solution. Once a reduced set of eigenvectors and eigenvalues is available, one can easily proceed to evaluate the dynamic response of the system due to dynamic loading as described in textbooks such as Ref. 17.

C. Reliability Analysis Using the Stochastic Finite Element Method (COSSAN)

The computation of the reliability of a structure in terms of the probability of failure generally requires the evaluation of the multidimensional integral[11]

$$p_f = \int_{D_f} f_X(\boldsymbol{x}) \, \mathrm{d}\boldsymbol{x} \tag{12}$$

where f_X denotes the joint probability density function of the vector of random variables $\boldsymbol{X}$. The integration has to be performed over the failure domain D_f, which is usually bounded by means of the limit state function,

$$g(\boldsymbol{X}) = 0 \tag{13}$$

By convention, $g(\boldsymbol{X})$ is defined as follows:

$$g(\boldsymbol{X}) = \begin{cases} \leq 0 & \text{failure} \\ > 0 & \text{safe state} \end{cases} \tag{14}$$

For small and simple, mostly academic, types of structural problems, it is often possible to analytically compute the limit state function. However, if the model consists of more than a few degrees of freedom or if it shows complex behavior, an explicit limit state function can be derived in very few cases only.[9] Usually, $g(\boldsymbol{X}) = 0$ can only be computed pointwise for such structures.

At this stage, the RSM may be advantageously employed to fit a suitable interpolation function through these points, which is then used instead of the actual limit state function to compute p_f. From the point of view of mathematical tractability, polynomials are the first choice. Because a linear function cannot account for interactions between two random variables and, therefore, possibly neglects significant contributions to the probability of failure, second-order polynomials are often used, i.e.,

$$\bar{g}(\boldsymbol{X}) = a + \sum_{i=1}^{n_{\text{rv}}} b_i x_i + \sum_{i=1}^{n_{\text{rv}}} \sum_{j=1}^{n_{\text{rv}}} c_{ij} x_i x_j \tag{15}$$

Lowercase letters represent realizations, e.g., x_i, of a random variable X_i, stored at location i in the random vector $\boldsymbol{X}$. To obtain all coefficients (except a) of the second-order polynomial (15) from a determined system of equations,

$$n_p = n_{\text{rv}} + \frac{n_{\text{rv}}(n_{\text{rv}} + 1)}{2} \tag{16}$$

interpolation points are necessary. Because the determination of the limit state points is by far the computationally most expensive task in the whole reliability analysis, the amount of computational effort may reach the magnitude of direct MCS if the number of random variables to be considered is high. However, if only a few random variables are important, the response surface method is a promising tool for reliability analysis. Its weak point may be seen in the fact that the error cannot be quantified without knowing the real limit state function. However, in most practical cases, it should be sufficiently smooth and convex to keep the errors low.

Consider now the matrix equation of motion for a nonlinear, dynamically excited structure,

$$\boldsymbol{M}\ddot{\boldsymbol{u}} + \boldsymbol{R}(\dot{\boldsymbol{u}}, \boldsymbol{u}) = \boldsymbol{F}(t) \tag{17}$$

which must be solved repeatedly to compute points on the limit state. In the most general case, the mass matrix $\boldsymbol{M}$, the vector of restoring forces $\boldsymbol{R}$, and the applied forces $\boldsymbol{F}$ include randomness. To account for structural randomness with spatial correlation, the SFEM which is based on the random field concept, is employed. Random fields are a suitable tool to model the spatially correlated stochasticity of structural parameters. A random field $H_{\text{rf}}(\xi, \eta, \zeta)$ is a continuous function in terms of the geometric coordinates ξ, η, and ζ, whereas the finite element method, by its principle, is a discrete method. Hence, the random field must also be discretized into a set of random variables H_i.

The integration point method allows such a discretization, which combines accuracy of representation with the advantages of point discretization methods,[4,18] i.e., the random field is discretized at the finite element integration points,

$$H_i = H_{\text{rf}}(\xi_i, \eta_i, \zeta_i), \qquad i = 1, \ldots, n_{\text{int}} \tag{18}$$

The correlation in space is described by the autocorrelation function R_{HH} of the random field. An important parameter of R_{HH} is the correlation length l_H (Ref. 19). If l_H, representing the measure for the spatial correlation, is sufficiently long, i.e., if the mutual dependency of two values at two random field points with a fixed distance is high, the midpoint method proves to be adequate.[4] In the latter case, n_{int} would be the number of considered elements.

Because discretization of a random field generally produces a large number of random variables, the efficiency of the application of the RSM requires a significant reduction of the number of random variables. To obtain uncorrelated Gaussian variables, a two-step transformation of the vector of original random variables, $\boldsymbol{H}$, with joint cumulative distribution function $F_{\boldsymbol{H}}(\boldsymbol{h})$ is employed. If

$\boldsymbol{H}$ is not normally distributed, the Nataf model[12,13] is used for a transformation to the correlated Gaussian space, i.e.,

$$Z_i = \Phi_G^{-1}\left[F_{H_i}(H_i)\right] \tag{19}$$

The application of this model is based on the assumption that $F_{\boldsymbol{H}}(\boldsymbol{h})$ can be described by its marginal distribution functions and that $\boldsymbol{Z}$ is jointly Gaussian. To perform a transformation to uncorrelated Gaussian random variables X_i, the correlation matrix $\boldsymbol{C}_{ZZ}$ is decomposed by the solution of a standard eigenvalue problem,

$$\boldsymbol{C}_{ZZ} = \boldsymbol{\Psi}_{ZS}\boldsymbol{C}_{SS}\boldsymbol{\Psi}_{ZS}^T \tag{20}$$

The correlation matrix $\boldsymbol{C}_{SS}$ is a diagonal matrix, where the diagonal elements, $C_{SS}(i,i)$, represent the variances of the random variables S_i. Arranging these elements according to their value, it can be seen that, except for very short correlation lengths, a few random variables S suffice to describe the fluctuations of the random field to a high degree of accuracy. Hence, random variables with relatively small variances may be disregarded in the further reliability analysis. After generating samples, the back transformation is then performed by considering the n_{rv} most important random variables with respect to the random field description by

$$\boldsymbol{z} = \boldsymbol{\Psi}_{ZS,n_{\text{rv}}}\boldsymbol{s}_{n_{\text{rv}}} \tag{21}$$

and if the original random variables are non-Gaussian, subsequently by

$$h_i = F_{H_i}^{-1}[\Phi_G(z_i)], \qquad i = 1, \ldots, n_{\text{int}} \tag{22}$$

If a random dynamic loading is applied to the structure, or if the structure exhibits nonlinear behavior, sensitivity analyses in terms of principal component analysis may be utilized to further reduce the number of random variables to be considered by comparing their influence on the response of the structure.[4] Despite identical variance, the influence of two random variables S_i and S_j on the response may be different due to amplification or weakening effects. These effects may be caused by the respective boundary conditions and the degree of similarity between the shape of the random field and, for example, the eigenvector shapes of a structure. For the static load case, however, an additional sensitivity analysis would require too much computational effort.

To determine points on the limit state function, which will be approximated by a second-order polynomial [Eq. (15)], two out of the n_{rv} random variables are considered at a time. All other random variables are kept at their mean value. After a limit state criterion is defined, the values of these two random variables are now adapted by means of a modified bisection method until the limit state is approached with sufficient accuracy. For every adaptation, the response of the system has to be evaluated by a stochastic finite element computation in the Monte Carlo sense; i.e., a random field realization is generated, the structural matrices are set up, and the resulting equation of motion is solved in a quasideterministic manner for this sample.

III. Numerical Examples and Results

A. General Remarks

Three typical examples from spacecraft structural design are shown in this section. The first example compares deterministic considerations with the much more sophisticated probabilistic analysis of a sandwich plate on which different items are mounted. The second example shows one of the probabilistic analyses with a small satellite structure. For both examples the engineering tool ASKA_RV[2] is utilized.

The third example shows an application of the SFEM in combination with the RSM. A polar-mounted fuel tank with thickness variations is presented. All calculations for this example are performed by means of the code COSSAN.[8]

B. Devices on a Sandwich Plate

Ten boxes containing either electronic devices or small experimental setups are accommodated on a sandwich plate with 1.0-m edge length. The model is shown in Fig. 1. One of the experiments contains a pump causing harmonic excitation at three different frequencies, each having an amplitude of 1.0 N in x and z directions (phase lag: 90 deg). Interface forces an accelerations in the z direction at the excitation source shall be calculated. Furthermore, z accelerations at interfaces of boxes 8, 9, and 10 are of interest.

Table 1 Assumed variations of the sandwich plate example

Input variable	Distribution	Mean	Coefficient of variation, %
Young's modulus face sheet	Uniform	2×10^{10} N/m^2	25
Edge support rotational stiffness	Normal	1×10^{4} Nm/rad	10
Young's modulus plate box 5	Uniform	6×10^{10} N/m^2	8
Beam inertia box 5	Normal	6×10^{-8} m^4	10
Beam inertia box 8	Normal	14×10^{-7} m^4	10
Beam inertia box 10	Normal	14×10^{-7} m^4	10

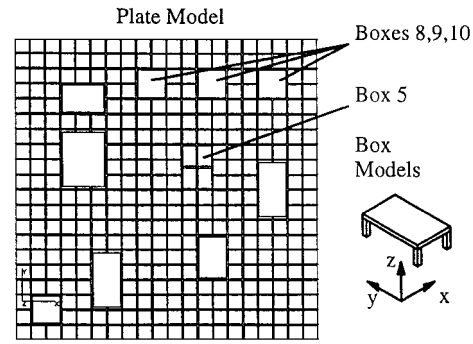

Fig. 1 Sandwich plate with 10 experiment boxes.

The first eigenfrequency of the sandwich plate is approximately at 40 Hz. Because of variations in the carbon fiber-reinforced plastics face sheet stiffness, this value may vary. In addition the rotational support stiffness at its edges is uncertain. All boxes have eigenfrequencies above 100 Hz. They are represented by simple models consisting of one plate and four beam elements (see Fig. 1). Variations in the beam properties are assumed for boxes 5, 8, and 10. The plate of box 5, accommodating the pump, is modeled using four plate elements. The stiffness uncertainty is reflected in the defined range for the Young's modulus of the plate. Finally, the excitation frequency may vary $\pm 2\%$. Therefore, three intervals are selected in the frequency response calculation: 65–67, 108–112, and 216–224 Hz. For each interval the response at the center and its boundaries is calculated. Eigenfrequencies are added if identified within the intervals. All results presented next correspond to the highest response value at three or more excitation frequencies in each interval. The assumed random parameters (the variations are obtained from Ref. 20) are listed in Table 1.

At the beginning of the study, an MCS with 250 simulations is performed. From this, using correlation plots, we evaluate which of the random variables affects which particular result. By doing so, it is found that because the box 5 plate has no eigenfrequencies below 100 Hz, its stiffness is not correlated with the interface force.

On the other hand, the sandwich plate stiffness has a significant influence (although the linear correlation coefficient c_xy is rather small). It can be seen in Fig. 2 that the extreme force at 66 Hz is strongly related to the face sheet stiffness of the sandwich plate. The largest values are found in the interval range in the marked region. However, the number of points in this interval is not sufficient yet to derive a force level with low failure probability, e.g., $p_f = 3 \times 10^{-3}$. Therefore another 200 simulations are performed using importance sampling. For the face sheet's Young's modulus a uniform sampling density function is selected in a limited interval from 12.9×10^9 to 14.8×10^9 N/m^2. Based on the additional simulation results and using as the failure condition the case that the interface force exceeds 2.39 N, the probability of failure can be estimated as 3.8×10^{-3} with a statistical error of 9.3×10^{-5}. In comparison, in a deterministic analysis using the mean values of the random variables, where the interface force is calculated as 1.78 N and then multiplied by a factor of safety of 1.5 (as is often used in spacecraft design), one would

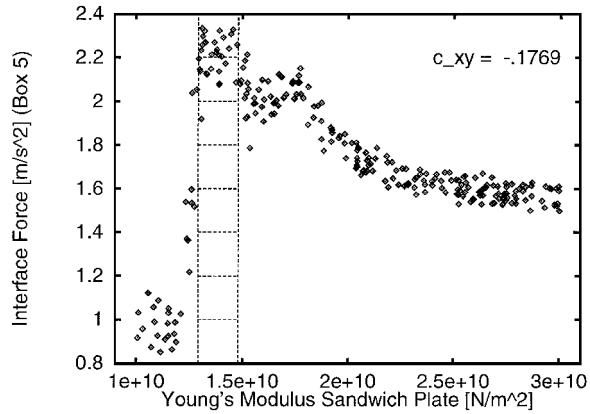

Fig. 2 Correlation of 66-Hz interface force and sandwich plate Young's modulus (box 5).

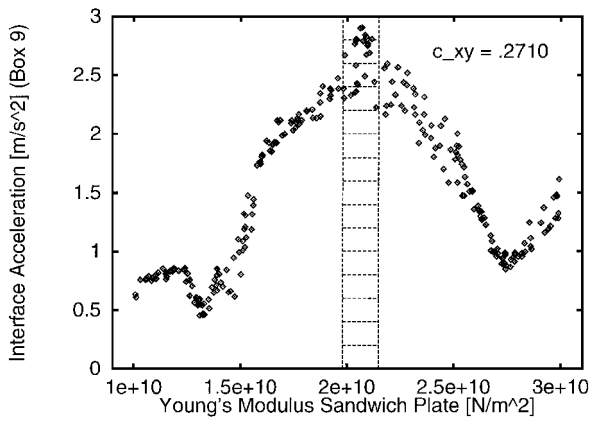

Fig. 3 Correlation of box 9 interface acceleration and sandwich plate Young's modulus.

identify a design interface force of 112% of the value obtained by importance sampling (2.39 N).

As mentioned earlier, the interface accelerations at the boxes 8, 9, and 10 are also of interest. Here it is found that at such a frequency, where responses depend very much on local modes, the accuracy of deterministic FE analyses is limited. The results from an MCS are discussed later.

Correlation plots as shown in Fig. 3 are typical for all boxes. It is found that the face sheet stiffness of the sandwich panel has a strong influence on the results. The waveform in the correlation plots is obtained because several resonance modes could shift towards the excitation frequency if the sandwich panel changes its stiffness. Figure 3 shows the extreme interface acceleration obtained at box 9. Again importance sampling is applied to calculate an acceleration level that is exceeded with a probability of 3×10^{-3}. Another 200 simulations are performed with a uniformly distributed face sheet's Young's modulus in an interval of 19.9×10^9 to 21.5×10^9 N/m^2. The desired acceleration level is found to be 2.87 m/s^2 with a probability of failure of $p_f = 2.98 \times 10^{-3}$ and a statistical error of 7.8×10^{-5}. Again the result is compared with a deterministic analysis using the mean values of the random variables. Here, the acceleration is 2.51 m/s^2, which when multiplied with a factor of safety of 1.5 yields 3.77 m/s^2 as a design acceleration.

Both examples indicate that the currently used factor of safety of 1.5 is rather conservative. However, the situation would change considerably if the nominal face sheet stiffness would drop to 70% of its original value. In the deterministic analysis the interface acceleration at box 9 would reduce to 0.74 m/s^2. If the uncertainty level would be maintained, a factor of safety would not be sufficient for a design with a low failure probability. This shows clearly that the currently used global safety factor in some cases may either over- or underestimate the response. A probabilistic analysis as shown here, however, may reflect the physical phenomenon more accurately. The overall computation time of 1.5 h for the simulation with 200 samples seems to be acceptable for the practicing engineer.

Table 2 Parts of the small demonstration spacecraft

Part	Material
Central cylinder	Honeycomb cylinder
Lower floor	Honeycomb plate
Upper floor	Honeycomb plate
Outer panels	Honeycomb plates
Solar arrays	Two stacks
Shear panels	Honeycomb plates

Table 3 Assumed variations of the satellite example

Input variable	Distribution	Mean, mm	Coefficient of variation, %
Face sheets central cylinder	Normal	0.60	10
Face sheets shear walls	Normal	0.30	3
Face sheets lower floor	Normal	0.50	10
Face sheets Y panels	Normal	0.30	10

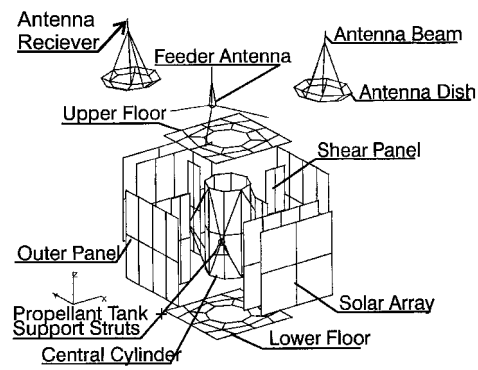

Fig. 4 Model of the small demonstration spacecraft.

C. Small Demonstration Spacecraft

As a second example a small demonstration spacecraft is used. Frequency response functions are evaluated, especially with the objective of identifying the reliability of the serviceability criteria of one of the antennas. The parts are similar to real spacecraft parts as listed in Table 2 and sketched in Fig. 4

Four propellant tanks are modeled as lumped masses attached on struts to the central cylinder. Furthermore secondary structural items such as antennas, reaction wheels, batteries, and electronic boxes are modeled. The dynamic behavior of this FE model is representative for this type of telecommunication satellite.

It is assumed that the applied random variables as listed in Table 3 are not correlated, as the production process as well as the materials differ from each other.

The excitation at the launcher interface is a prescribed sinusoidal acceleration of 1 g. The frequency response is calculated within the range 1–50 Hz. A modal damping coefficient of 2% is introduced for all modes.

The condition of failure shall be the loss of the serviceability of an antenna. This occurs if the local acceleration of the receiver is higher than 115 g. The deterministic analyses show that this event may happen in the frequency range of 45–50 Hz. Therefore the maximum acceleration in this range has been chosen as a response request.

In a first step a direct MCS is performed, where all mentioned variables were used. The results show a strong correlation of the face sheet thickness of the lower floor with the maximum amplitude, whereas the other random variables seem to have no influence. However, it can be seen in Fig. 5 that the failure event occurs in the tail region of the distribution of the maximum acceleration. Therefore the sample size is insufficient, and the importance sampling technique has to be applied. The sampling mean value of the face sheet thickness distribution of the lower floor is shifted to 0.6 mm. This value has been selected from correlation plots. The standard deviation of the sampling is changed to 0.01 mm (it was 0.05 mm before). The result of this procedure is shown in Fig. 6. (Note that the $>$ signs denote the 115-g mark.) The probability that the

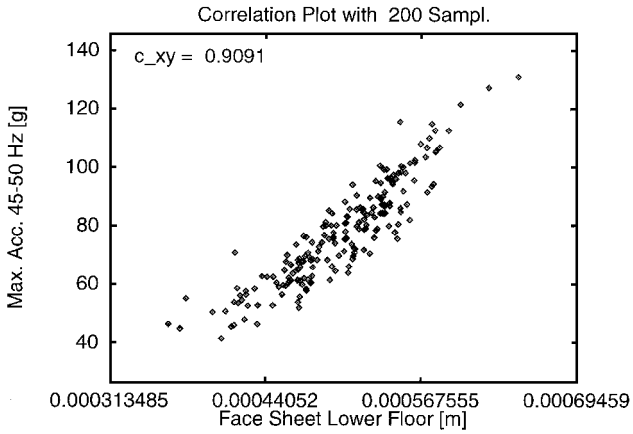

Fig. 5 Correlation of the lower floor face sheet thickness with the extreme acceleration at the receiver.

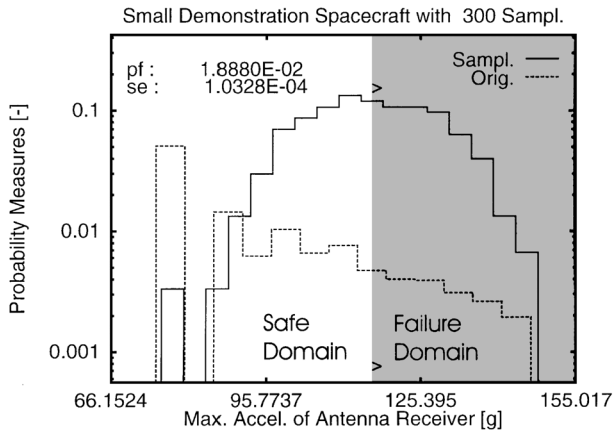

Fig. 6 Result of the small demonstration spacecraft analysis.

acceleration of the receiver exceeds 115 g can then be estimated to $p_f = 1.9 \times 10^{-2}$ with a statistical error of 0.5%.

Finally, note that the straightforward simulation procedure for the solution of this problem takes about 7 h for a sample size of 200. However, by applying the reduction technique as presented in Sec. II.B, the computation time can be reduced to 1 h with similar accuracy.

D. Fuel Tank Model

Finally, a typical example for the application of random fields in the field of spacecraft design is the model of a fuel tank exposed to internal pressure. A model of a fuel tank as shown in Fig. 7 is considered for the following numerical studies. The tank consists of three parts, i.e., two spherical parts with radius of 0.5 m and wall thickness $t_s = 1$ mm, which are connected by one cylindrical part with radius of 0.5 m, length of 0.2 m, and wall thickness $t_c = 2$ mm. The model is discretized using three-node shell elements with six degrees of freedom per node and 26 integration points.[8] Each half-sphere is modeled with 900, the cylinder with 1536 elements. The model has 10,020 active degrees of freedom. The tank is polar mounted. Axial expansion is not suppressed.

The load on the tank is assumed to be caused by internal pressure. It is assumed that the tank is made of titanium, with a Young's modulus of 1.052×10^{11} N/m^2, a Poisson ratio of 0.33, and a yield stress of 3.9×10^8 N/m^2.

During this investigation, the only parameter that is considered to vary randomly is the tank wall thickness. At this stage the uncertainties of all other parameters, like Young's modulus or the yield stress, are neglected. The uncertain thickness of the model is expressed by the application of random fields, exemplified in Fig. 7 for the cylinder part. Note that the values of the random field sample are shown in an amplified form and added to the nodal coordinates in terms of displacements perpendicular to the surface of the tank. This has

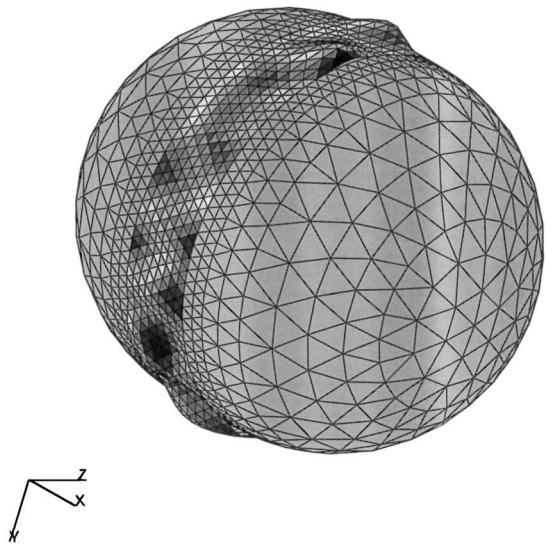

Fig. 7 FE model of a polar-mounted fuel tank. A random field is visualized as displacements on the cylinder part.

been done using a special kind of random field for visualization purposes only.

Three different, statistically dependent, isotropic, homogeneous and zero mean random fields are used, one for the cylinder part and one for each half-sphere. The dependence is such that one random variable influences all fields at the same time corresponding to the production process, where all parts are produced by the same machine. For instance, the variability of the wall thickness of the cylinder is described by

$$t_c(\xi, \eta, \zeta) = t_{0,c}[1 + H_c(\xi, \eta, \zeta)] \tag{23}$$

where $t_{0,c}$ is the mean value and H_c the random field defined for the elements of the cylinder. An autocorrelation function of an exponential type is used to model the spatial correlation of the random fields, i.e.,

$$R_{HH} = \sigma_{HH}^2 \exp\left(-\frac{\|\boldsymbol{\xi}\|}{l_H}\right) \tag{24}$$

where

$$\|\boldsymbol{\xi}\| = \sqrt{(\xi_i - \xi_j)^2 + (\eta_i - \eta_j)^2 + (\zeta_i - \zeta_j)^2} \tag{25}$$

is the absolute distance between two points of a random field.

Each random field is characterized by a coefficient of variation of 1%, i.e., a standard deviation of $\sigma_{HH} = 0.01 \times t_0$ and a correlation length of $l_H = 0.5$ m. The midpoint method was used because the correlation length is sufficiently large in comparison to the element size. With this correlation length and for the given FE model, the three most important uncorrelated Gaussian random variables, as obtained by means of the transformations (19) and (20), account for approximately 60–70% of the random field's fluctuations (see Fig. 8).

As the limit state criterion, the exceeding of the so-called maximum acceptable stress equal to the deterministic yield stress is chosen. Using this model, one can calculate the reliability of the structure, conditional on the assumption of randomness only in the thickness of the tank. In a first investigation with three random variables, the pressure was varied until a target failure probability of $p_f \approx 10^{-6}$ or lower was reached. During this investigation it was found that the failure probability is quite sensitive to the loading. Meaningful results were then obtained for an internal pressure of $p_i = 12.15 \times 10^5$ Pa. Using this value for the deterministic computation as well as the mean values of the thickness, a maximum stress of $S = 3.74 \times 10^8$ N/m^2 results for the given loading conditions. This value is only 4.2% below the yield stress of the material.

Depending on the number of important uncorrelated Gaussian random variables that are considered for determination of points on the limit state function, the resulting probability of failure varies

Table 4 Probability of failure vs number of random variables (RVs) per random field and CPU time approximately required on a workstation HP 9000 735

Number of RVs:	2	3	4	5
CPU time, days	——	1.9	2.8	4.5
$p_f \times 10^{-7}$	1.39	1.82	2.12	3.10
Statistical error, %	2.0	3.5	3.6	2.8
Number of points	5	9	14	20

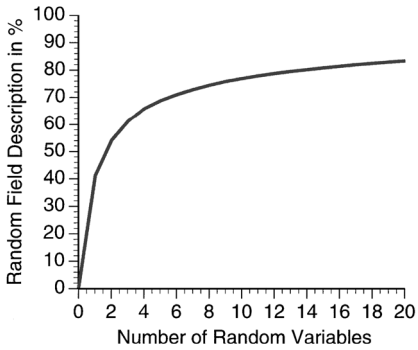

Fig. 8 Percentage of random field description vs number of random variables.

between 1.39 and 3.1×10^{-7}. However, all calculations yield the same order of magnitude. Table 4 gives an overview of the results using a different number of random variables per random field.

To compute one point on the limit state, on a workstation of type HP 9000 735, approximately 5 h of CPU time were required. The time is required mainly for solving repeatedly the stochastic finite element equation system. In contrast to this, the need of CPU time for the application of the response surface and the calculation of the failure probability is negligible. Hence, the analysis with five random variables takes more than four days.

Finally, note that the high computation time is caused by the size of the system matrices and not by the application of the random field model. However, the discretization of the random field models demands almost the same portion of storage capacity of the computer as the FE model itself. Hence, for practical purposes the number of random fields in a model as well as the number of important random variables that reflect them must be considered carefully.

IV. Conclusions

Two different approaches toward modeling of uncertainty applied to spacecraft structures are shown in this study. Based on the results, the following conclusions are obtained.

The first approach is to use MCS techniques for the stochastic analysis of the structure. Direct MCS is applied to obtain some information on the correlation between the applied random variables and the structural response and thus to identify important regions where failure could occur. The results of the examples show that the obtained correlation structures cannot be reflected well by correlation coefficients to automatically evaluate important regions. In the identified important region a manually adjusted importance sampling procedure is performed to calculate failure probabilities. Moreover, a concept is described to reduce the computational effort in case of large linear structures. Two examples are presented for this approach (in this case both are analyzed using ASKA_RV; however, the approach is easy to extend to other types of FE packages). 1) A sandwich panel with different electronic boxes and experiments is applied to compare deterministic reasoning with a probabilistic analysis. 2) One part of the dynamic analysis of a small demonstration spacecraft is presented. It is found that this approach is convenient to be applied by design engineers and at the same time is extremely versatile due to its simplicity.

The second approach utilizes the response surface method to describe the limit state function between the safe domain and the failure domain. This method in combination with the random field approach of stochastic finite elements allows for a good estimation of failure probabilities. The methods are integrated in the software environment COSSAN. This is basically an all-in-one software, i.e., it allows for direct interaction between different fields of analysis, e.g., FE and probabilistic parts, respectively. Furthermore, COSSAN has also a user interface that enables direct access to numerical values during the analysis. As an example the model of a fuel tank with internal pressure is treated.

Acknowledgments

This work is partially supported by the European Space Agency, Contract 11810/96/NL/JG, and the Austrian National Space Program, and that support is gratefully acknowledged by the authors.

References

[1]Pradlwarter, H. J., Diez, R., Klein, M., and Schuëller, G. I., "On Engineering Tools for Numerical Evaluation of Structural Scatter and Reliability," *Proceedings of the International Conference on Spacecraft Structures and Mechanical Testing*, Cépaduès-Editions, Paris, France, 1994, pp. 695–708.

[2]CERRAM/LASSO, "Stochastic Structural Analysis ASKA_RV Version 2.0-User's Manual," CERRAM-Munich, LASSO-Stuttgart, Germany, 1995.

[3]"ASKA Version 9.2," IKO Software Service Gmbh, Stuttgart, Germany, 1993.

[4]Brenner, C. E., "Ein Beitrag zur Zuverlässigkeitsanalyse von Strukturen unter Berücksichtigung von Systemunsicherheiten mit Hilfe der Methode der Stochastischen Finiten Elemente," Dissertation, Fakultät für Bauingenieurwesen und Architektur, Leopold-Franzens Univ., Innsbruck, Austria, 1995 (in German).

[5]Ghanem, R., and Spanos, P., *Stochastic Finite Elements: A Spectral Approach*, Springer–Verlag, New York, 1990.

[6]Shinozuka, M., *Stochastic Mechanics*, Vols. 1–3, Dept. of Civil Engineering and Engineering Mechanics, Columbia Univ., New York, and Dept. of Civil Engineering and Operational Research, Princeton Univ., Princeton, NJ, 1987–1988.

[7]Vanmarcke, E., *Random Fields: Analysis and Synthesis*, MIT Press, Cambridge, MA, 1983.

[8]"COSSAN (*Co*mputational *S*tochastic *S*tructural *A*nalysis)—Stand-Alone Toolbox, User's Manual," IfM-Nr: A, Inst. of Engineering Mechanics, Leopold-Franzens Univ., Innsbruck, Austria, 1996.

[9]Schuëller, G. I., and Bayer, V., "Computational Procedures in Structural Reliability," *International Symposium on Uncertainty Modeling and Analysis (ISUMA)*, edited by B. M. Ayyub, IEEE Computer Society Press, Los Alamitos, CA, 1993, pp. 552–559.

[10]Hammersley, J. M., and Handscomb, D. C., *Monte Carlo Methods*, Methuen & Co. Ltd., London, 1967.

[11]Schuëller, G. I., and Stix, R., "A Critical Appraisal of Methods to Determine Failure Probabilities," *Journal of Structural Safety*, Vol. 4, 1987, pp. 293–309.

[12]Liu, P.-L., and Der Kiureghian, A., "Multivariate Distribution Models with Prescribed Marginals and Covariances," *Probabilistic Engineering Mechanics*, Vol. 1, No. 2, 1986, pp. 105–112.

[13]Nataf, A., "Détermination des distributions de probabilités dont les marges sont données," *Comptes Rendus de l'Académie des Sciences*, Vol. 225, 1962, pp. 42, 43.

[14]Maess, M. A., Breitung, K., and Geyskens, P., "Asymptotic Importance Sampling," *Proceedings of 6th Specialty Conference on Probabilistic Mechanics and Structural Design and Geotechnical Reliability*, edited by Y. K. Lin, American Society of Civil Engineers, New York, 1992, pp. 96–99.

[15]"ISPUD—Importance Sampling Using Design Points, User's Manual," IfM-Nr: C, Inst. of Engineering Mechanics, Leopold-Franzens Univ., Innsbruck, Austria, V 3.0, V 4.0, 1997.

[16]Székely, G. S., Pradlwarter, H. J., Klein, M., and Schuëller, G. I., "Numerical Evaluation of the Variation of the Response of Dynamical Systems Due to Parameter Uncertainties," *Proceedings of ESREL*, edited by I. A. Watson and M. P. Cottam, Vol. 1, Chameleon Press, England, 1995, pp. 203–215.

[17]Clough, R. W., and Penzien, J., *Dynamics of Structures*, 2nd ed., McGraw–Hill, New York, 1993.

[18]Brenner, C. E., and Bucher, C., "A Contribution to the SFE-Based Reliability Assessment of Nonlinear Structures Under Dynamic Loading," *Probabilistic Engineering Mechanics*, Vol. 10, 1995, pp. 265–273.

[19]Schuëller, G. I., and Brenner, C. E., "Stochastic Finite Elements—A Challenge for Material Science," *Advances in Building Material Science*, edited by A. Gerdes, Aedificatio Publ., Freiburg, Germany, 1996, pp. 259–273.

[20]"Dimensioning of Spacecraft Structures for Transient Flight Events and Associated Testing," Aerospatial Rept., Doc. SE/TMA/5532, Ed. 1, Aug. 1993.

A. D. Belegundu
Associate Editor